STUDIES ON IRREGULAR GRAPH

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Abstract - ***In any graph G, two vertices in G must have the same degree, so an irregular graph cannot be defined as a graph with all vertices of different degrees. A graph is thus locally irregular if for each vertex v of G the neighbours of v have distinct degrees, and these graphs are thus termed highly irregular graphs. In this paper we obtain neighbourly irregular graph, neighbourly irregular graph products, highly irregular and maximally k-path irregular graphs.***

I. INTRODUCTION

There are many notations of irregular graphs which arise in the literature. In this dissertation, we study about some results in irregularness of graphs. we collect some definitions on graphs which are needed for the subsequent chapters.

We discuss about the neighbourly irregular graph related into gracefulness, ply number, clique graphs and minimal edge covering. we discuss about some products of neighbourly irregular graph. we discuss about the highly irregular graph and highly irregular bipartite graphs. We discuss about the upper bounds on the size of maximally irregular graphs, maximally irregular triangle-free graphs and also discuss about induced subgraph in a k-path irregular graphs.

II. NEIGHBOURLY IRREGULAR GRAPH

In this chapter we suggests a method to construct a neighbourly irregular graph of order n and also this chapter includes a few facts possessed by these neighbourly irregular graph. Construct the neighbourly irregular graph related to the gracefulness, ply number and lace number, clique graphs and minimal edge covering are also investigated. All the results presented in this chapter are taken from [1].

Definition 2.1:

A connected graph G is said to be **neighbourly irregular graph** if no two adjacent vertices of G have the same degree.

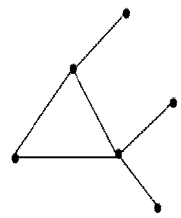

Figure 1

Theorem 2.2:

If v is a vertex of maximum degree in a neighbourly irregular graph then at least two of the adjacent vertices of v have the same degree.

Theorem 2.3:

$K_{m,n}$ a neighbourly irregular if and only if $m \neq n$.

Theorem 2.4:

Let G be a neighbourly irregular graph of order n then for any positive integer $m < n$, there exist at most m vertices of degree $(n - m)$.

Theorem 2.5:

If a graph G is neighbourly irregular then no P_4 contains internal vertices of same degree in G.

Theorem 2.6:

If a graph G is neighbourly irregular then G^C is not neighbourly irregular graph.

Theorem 2.7:

Any graph of order n can be made to be an induced sub graph of neighbourly irregular graph of order at most $(n+1)C_2$.

Theorem 2.8:

Let a positive integer n and a partition $(n_1, n_2, \ldots, n_k)$ of n such that all n_i's are distinct, there exists a graph of order n and size, $\frac{1}{2}\{n^2 - (n_1^2 + n_2^2 + \cdots + n_k^2)\}$ and this graph is neighbourly irregular.

Theorem 2.9:

Let n be a positive integer and (n_1, n_2, n_3) be a partition of n with distinct parts and $n_1 = 1$ then $NI_{(n_1,n_2,n_3)}$ is graceful.

Theorem 2.10:

Let n be any positive integer and $(n_1, n_2, \ldots, n_k)$ be any partition of n with distinct parts then the ply number of $NI_{(n_1,n_2,\ldots,n_k)}$ is $n - n_k$.

Theorem 2.11:

The clique graph of $NI_{(n_1,n_2,\ldots,n_k)}$ is a m–regular graph of order $n_1, n_2, \ldots, n_k$ and of size $\frac{mn_1n_2\ldots n_k}{2}$ where $m = n_1n_2 \ldots n_k - (n_2-1)(n_3-1) \ldots (n_k-1) - 1$. If $n_1 = 1$ then its clique graph is the complete graph k_l where $l = n_1, n_2, \ldots, n_k$.

Theorem 2.12:

A minimal covering edge family of the $NI_{(n_1,n_2,\ldots,n_k)}$ graph has the cardinality $n_1 + n_3 + \cdots + n_k$ if k is odd or $n_2 + n_4 + \cdots + n_k$ if k is even.

III. NEIGHBOURLY IRREGULER GRAPH PRODUCTS

In this chapter we obtain neighbourly irregular subdivision graphs, line graphs and total graphs. Given a positive inter n and a partition of n with distinct parts, this chapter suggests a method to construct a neighbourly irregular graph of order n. We consider finite, simple, connected graphs. Let G be a graph with n vertices and m edges. The vertex set and edge set of G are denoted by V(G) and E(G) respectively. Let $deg_G(v)$ denote the degree of a vertex v in G. The neighbourly irregularity of some graph products is also investigated. We now present some graph operations that will be used in this chapter. Let G and H be two graphs. The join G + H of graphs G and H with disjoint vertex sets V_1 and V_2 and edge sets E_1 and E_2 is the graph union G∪H together with all the edges joining V_1 and V_2. All the results presented in this chapter are taken from [6].

Theorem 3.1:

Let G be a graph. The subdivision graph S(G) is neighbourly irregular graph if and only if G does not have any vertex of degree two.

Theorem 3.2:

For any graph G, its line graph L(G) is neighbourly irregular graph if and only if N(u) contains all vertices of different degree for all $u \in$ V(G).

Theorem 3.3:

If G is neighbourly irregular graph then L(G) is not neighbourly irregular graph.

Theorem3.4:

For each integer $k \geq 1$ there exists a graph G with maximum degree Δ(G) = K such that L(G) is neighbourly irregular graph.

Theorem 3.5:

For any graph G then the total graph T(G) is neighbourly irregular graph if and only if both G and L(G) are neighbourly irregular graphs.

Theorem 3.6:

For any two graphs G and H, the Cartesian product G × H is neighbourly irregular graph if and only if both G and H are neighbourly irregular graphs.

Theorem 3.7:

For any neighbourly irregular graph G and a regular graph H then the tensor product G ⊗ H is neighbourly irregular graph.

Theorem 3.8:

Let G and H are graphs with vertex degree sets S_G and S_H respectively. Then G + H is neighbourly irregular graph if and only if both G and H are neighbourly irregular graphs and(|V(G)| = |V(H)| and $S_G \cap S_H = \phi$) or (|V(G)| – |V(H)| = $t > 0$ and $S_G \cap \{ a_i + t / a_i \in S_H \} = \phi$).

Theorem 3.9:

Let G and H be nontrivial graphs. Then G o H is neighbourly irregular graph if and only if both G and H are neighbourly irregular graphs.

Theorem 3.10:

Let G and H be nontrivial graphs. Then G [H] is neighbourly irregular graph if and only if both G and H are neighbourly irregular graphs.

Theorem 3.11:

Let G and H be nontrivial graphs and let U be a nonempty subset of V(G). Then G(U) ⊓ H is neighbourly irregular graph if and only if H is neighbourly irregular graph and for each $gg' \in E(G)$ and $v \in V(H)$,

$$deg_G(g_2) - deg_G(g_1) \neq \begin{cases} 0 & \text{if } g_1, g_2 \in U \text{ or } g_1, g_2 \in V(G) - U \\ deg_H(v) & \text{if } g_1 \in U \text{ and } g_2 \in V(G) - U \end{cases}$$

Theorem 3.12:

If G and H are neighbourly irregular non trivial graphs and there are no edge $g_1g_2 \in E(G)$, $h_1h_2 \in E(H)$ such that $deg_G(g_1) = deg_H(h_1)$ and $deg_G(g_2) = deg_H(h_2)$ then G ⊠ H is neighbourly irregular graph.

IV. HIGHLY IRREGULAR GRAPHS

In this chapter we investigate several problems concerning the existence and enumeration of highly irregular graphs and suggest a method to construct a neighbourly irregular graph of order *n*. we consider only finite, simple, connected graphs and we establish some result on highly irregular bipartite graphs. Throughout this chapter $H_{n,n}$ denotes the highly irregular bipartite graph with bipartite sets X = $\{v_1, v_2, \ldots, v_n\}$ and Y = $\{u_1, u_2, \ldots, u_n\}$ and with edge set E = $\{v_iu_j: 1 \le i \le n,\ n - i + 1 \le j \le n\}$. All the results presented in this chapter are taken from [3, 7].

Definition:

A connected graph G is said to be **highly irregular** if each neighbour of any vertex has different degree.

Theorem 4.1:

Every graph of order $n \geq 2$ is an induced sub graph of a highly irregular graph of order $4n - 4$.

Theorem 4.2:

The size of a highly irregular graph G of order $2n + 1$, $n \geq 4$ is at most, $n(n + 1)/2 + \left\lfloor\frac{(n+1)}{5}\right\rfloor$.

Theorem 4.3:

For any positive integer $n \neq 3$, 5 or 7, there is a highly irregular bipartite graph of order *n*.

Theorem 4.4:

For any two integer *n* and *m*, $n \geq m \geq 3$, there is a highly irregular bipartite graph with maximum degree *m* and with bipartite sizes (*n, n*).

Theorem 4.5:

The maximum degree of a highly irregular bipartite graph with bipartite size (*n, m*), $n < m$ is at most $n - 1$.

Theorem 4.6:

For any *n* and *m*, $3 \leq n \leq m \leq n + \binom{n}{2} - 1$, there is a highly irregular bipartite graph with bipartite size (*n, m*).

Theorem 4.7:

Every bipartite graph with bipartite size (*n, m*), $n \geq m \geq 3$ is an induced sub graph of a highly irregular bipartite graph with bipartite size $(2(n + m) - 2,\ 2(n + m) - 2)$.

V. MAXIMALLY AND K-PATH IRREGULAR GRAPHS

In this chapter we consider upper bounds on the size of maximally irregular graphs and maximally irregular triangle-free graphs. Further, we will establish asymptotically tight upper bounds on the size of maximally irregular graphs and maximally irregular triangle- free graphs in terms of order. For a vertex *u* of G we will denote the open neighbourhood of *u*, i.e., the set of all vertices adjacent to *u*, by N(*u*), and the closed neighbourhood of u, i.e., the set N(*u*) ∪ {*u*}, by N[*u*]. For a subset S ⊆V(G), we will denote by G[S] the sub graph of G induced by S. Also this chapter, we investigate for every graph G and every odd positive integer *k* that G can be embedded as an induced sub graph in *k*-path irregular graphs. The highly irregular graphs are precisely the 2-path irregular graph, while the totally segregated graphs are the 1-path irregular graphs. We present some results concerning *k*-path irregular graphs in this chapter. All the results presented in this chapter are taken from [4, 8].

Definition 5.1:

The connected graph G is said to be maximally irregular if t(G) = Δ(G) where t(G) is irregularity index and Δ(G) is the maximum degree of the graph.

Definition 5.2:

A connected graph G is k-path irregular, $k \geq 1$, if every two vertices of G that are connected by a path of length k have distinct degrees.

Theorem 5.3:

Every highly irregular graph is maximally irregular graph.

Theorem 5.4:

Every highly irregular graph G of order $n \geq 2$ and maximum degree Δ is an induced sub graph of a maximally irregular graph H of order $n + \Delta + 1$. Moreover, the graph H is not highly irregular.

Theorem 5.5:

Let G be a maximally irregular graph of order n. then the size m of G satisfies, $m \leq \frac{(n+2)(n-1)}{4}$. Moreover, the coefficient of n^2 is best possible.

Theorem 5.6:

Let G be a maximally irregular graph of order n, (G) ≥ 1. Then the size m of G satisfies,

$$m \leq \begin{cases} \frac{n^2-2n}{4} + \frac{2n-\delta(G)}{2}.\delta(G), & n(\mathrm{mod}\ 2) \equiv \delta(G)(\mathrm{mod}\ 2) \\ \frac{n^2-2n}{4} + \frac{2n-\delta(G)}{2}.\delta(G) + \frac{1}{4}, & \text{Otherwise} \end{cases}$$

VI. CONCLUSION

Graph theory deals with the study of problems involving discrete arrangements objects, where concern is not with the internal properties of the objects but the relationship among them. In this thesis we have studied the irregular graphs. In any graph G ,two vertices in G must have the same degree. so an irregular graph cannot be defined as a graph with all vertices of different degrees.

Irregular graphs are useful. We have found four distinct graph invariants used to measure the irregularity of a graph. Then we proved some important definitions and properties of neighbourly irregular , highly irregular, maximally and k-path irregular graphs.

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