A Study on Split Domination Number of a Graph

R.Sheela Navarose[#1],C.Chithirai Selvi[#2]M.Sc., M.Phil.,
[1]*Research scholar, Department of mathematics, Prist university Madurai campus, India*
[2]*Asisstant Professor, Department of mathematics Prist university Madurai campus*

Abstract

In this paper we have briefly study about the split domination number of graphs .A graph is a graph in which the vertices can be partitioned in to a clique and an independent set.

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I INTRODUCTION

In this we have discuss about the split domination number of graphs.kulli and Janakiram was introduced and explained about the term split domination number of graphs then we here see about the strong split domination number of graphs and total strong split domination number of graphs

II PRELIMINARIES

DEFINITION 2.1

If two vertices are incident with an edge,then they are called adjacent vertices.

DEFINITION 2.2

If two edges are incident with a common vertex then they are called **adjacent edges.**

DEFINITION 2.3

The order of G is the cardinality of $V(G)$ and is denoted by $|V(G)| = p$.

The order of G is the cardinality of $E(G)$ and is denoted by $|E(G)| = q$.

DEFINITION 2.4

A graph in which each vertex has the same degree is a regular anular graph.

DEFINITION 2.5

A graph G is said to be **K-regula**r if $deg(v)=K$, for every $v \in V(G)$

DEFINITION 2.6

A regular graph of degree three is called a **cubic graph**.

DEFINITION 2.7

The complete bigraph $k_{1,n}$is called a star.

DEFINITION 2.8

A Zero-regular graph is called a **null graph** and is denoted by O_n.

DEFINITION 2.9

A graph *G* is said to be complete if every two vertices of *G* are adjacent. The complete graph on *P* vertices is denoted by K_p.

DEFINITION 2.10

A graph *H* is called a **subgraph**of *G if* $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$ and we write $H \subseteq G$

DEFINITION 2.11

If $H \subseteq G$, $H \neq G$ is called **proper subgraph**of *G*.

DEFINITION 2.12

If $V(H) = V(G)$ thenH is called a **spanning subgraph**of *G*.

DEFINITION 2.13

Let *G* be a graph and $V \subseteq V(G)$.

Let H' be a subgraph of *G* with vertex set V' and the edges of H' are those edges of *G* which have both ends in V'.Then$H'$is called in inducedsubgraph of *G* induced by V' and is denoted by$\langle V' \rangle$

DEFINITION2.14

If *e* is an edge of *G* and *u, v* are vertices of *G*. such that $\varphi_G(e)=uv$.

Then u and v are called **end vertices** of *e* and *e* is said to be incident with *u* and *v*.

DEFINITION 2.15

Two vertices *v* and *w* of graph *G* are adjacent if there is an edge *vw* joining them and the vertices *v* and *w* are incident with such an edge.Similarly two distinct edge *e* and *f* are adjacent if they have a vertex in common.

DEFINITION 2.16

A **bipartite graph** is one whose vertex set can be partitioned into two subsets *X* and *Y*.so that each edge has one end in *X* and one end is*Y*. Such a partition(X, Y) is called a bipartition of a graph.

DEFINITION 2.17

If every vertex in *X* is adjacent to all vertices in *Y*. Then it is called a complete bipartite graph.

If$|X| = m$ and$|Y| = n$

Such a graph is denoted by $K_{m,n}$.

DEFINITION 2.18

Let *G* be a graph $v \in V(G)$.Then the degree of a vertex *v* is the number of edges incident with *v* and is denoted by $deg(v)$.

DEFINITION 2.19

Let *G* be a graph. An alternating sequence of vertices and edges starting with a vertex and ending with a vertex and is such that every edge in the sequence is incident on the preceding and succeeding vertices is called a walk in the graph. It may be denoted by *W*.

DEFINITION 2.20

A walk in which vertex appears more than once is called a **cycle**.

DEFINITION 2.21

If the edge is distinct, but the vertices are not distinct, the sequence of edges is called a **trail.**

DEFINITION 2.22

A walk is called atrail if all its edges are distinct.

DEFINITION 2.23

A distance$d(u, v)$ between two vertices u and v in a connected graph$G$is the length of any shortest path joining u and v.

DEFINITION 2.24

The diameter of a connected graph $G$is the maximum distance between two vertices of $G$and is denoted by $diam(G)$.

DEFINITION 2.25

A graph G is said to be acyclic if it contains no cycle.A connected acyclic is called a tree.In a tree $p = q - 1$.

DEFINITION 2.26

A Star is a tree with exactly one (respectively two) vertices of degree greater than one.

DEFINITION 2.27

A graph $G$is said to be connected if any two vertices of G are joined by a path. A graph $G$is said to be disconnected if any two vertices of $G$are not joined by a path.

A maximal connected subgraph of G is called a component of G

DEFINITION 2.28

A maximum complete graph of $G$is called a clique of G.

The order of a maximum clique of $G$is the clique number of G and is denoted by $W(G)$.

DEFINITION 2.29

A vertex $v$of a graph $G$is a **cut vertex** of G if $K(G - V) > K(G)$.

DEFINITION 2.30

A subset $S$of vertices of graph G is said to be **independent** if any two vertices in $S$are not adjacent in G.

The vertex **independence number**$\beta_0(G)$is the maximum cardinality among two independent sets of vertices of *G*.

DEFINITION 2.31

A subset *K*of vertices of a graph *G*is called **covering** of *G*. If every edges of *G*has at least one end in *K*.

DEFINITION 2.32

Let $G = (V, E)$ be a graph. A graph $D \subseteq V$ is a dominating set of *G*. if every vertex in $V-D$ is adjacent to some vertex in *D*.

The Domination number $\gamma(G)$ of *G* is the minimum cardinality of a dominating set.

III SPLIT DOMINATION NUMBER OF A GRAPH

THEOREM 3.1

For any graph G,$v_s(G) \leq \alpha_0(G)$ ………………(1)

Where $\alpha_0(G)$ is the vertex covering number of G.

Proof:

Let S be a maximum independent set of vertices in G.Then S has at least two vertices and every vertex in S is a adjacent to some vertex in V\S.This implies that V\S is a dominating set of G.

We state without proof a straight forward result that chatacterizes dominating set of G that are split dominating sets.

THEOREM 3.2

A split dominating set D of G is minimal if and only if for each vertex v∈D on of the following is satisfied :

(i) there exists a vertex $u \in V\backslash D$ such that $N(u) \cap D = \{v\}$

(ii) v is an isolate in ‹D›

(iii) (V\D) ∪ {V}

Proof:

Suppose D is minimal and there exists a vertex v ∈D such that v does not satisfy any of the above conditions. Then by condition (i) and (ii) ,D’ = D\{V} is a dominating set of G.Also by (iii),<V\D> is disconnected.This implies that D’ is a split dominating set of G a contradiction

The converse isobivious ,

Now we obtain another upper bound on $V_S(G)$

THEOREM 3.3

If G has one cut vertex v and at least two blocks H_1 and H_2 with v adjacent to all vertices of H_1 and H_2 then v is in every V_S-set of G

Proof:

Let D be a V_S-set of G. suppose v $\in$ V\D.

Then each of H_1 and H_2 contributes at least one vertex to D say u and w respectively

This implies that D'=D\{u,v}∪{v} is a split dominating set of G,

Which is the contradiction

Hence v is in everyV_S –set of G.

IV STRONG SPLIT DOMINATION NUMBER OF A GRAPH

THEOREM 4.1

If a graph G has independent strong non split dominating set. Then diam (G) $\leq$ 3

Where diam (G) is the diameter of G

Proof:

Let D be an independent strong non split dominating set of G

We consider the following cases

Case (i): Let $u \in V - D$

Then,d(u,v)=1

diam(G)$\leq$ 3.

Case(ii):

Let $u \in D$ and $v \in V - D$

Since D is independent

$\exists$a vertex $w \in V - D$,such that u is adjacent to w.

Thus, d(u,v)≤d(u,w)+d(w,v)

d(u,v)≤2

$d(u,v) \leq 3$

Case(iii):

Let $u, v \in D$

Since D is independent,∃ a vertices w_1 ,$w_2 \in$ V – D,such that u is adjacent to w_1 and v is adjacent to w_2

Thus, $d(u,v) \leq d(u,w_1) + d(w_1,w_2) + d(w_2,v)$

$d(u,v) \leq 1+1+1$

$d(u,v) \leq 3$

Thus for any vertex $u,v \in V$

$d,(u,v) \leq 3$

THEOREM 4.2

Let G be a graph satisfying the following conditions.

(i) $W(G) \geq 3$
(ii) diam $(G) \geq 3$

Then $\gamma_{ss}(\bar{G}) \leq \gamma_{ss}(G)$

Proof:

Let D be a γ_{sns} – set of G

Since $\gamma_{sns}(G) \leq P - W(G) + 1$ and given $W(G) \geq 3$

$\gamma_{sns}(G) \leq P - 2$

D has at least P – 2 vertices

⟹ V – D has at least 2 vertices

Given, diam $(G) \geq 3$

Every vertex in V – D is not adjacent to at least one vertex in D of G

Every vertex in V – D is adjacent to at least one vertex in D of $\bar{G}$

D is a dominating set of $\bar{G}$

In $\bar{G}$, $\langle V - D \rangle$ is totally disconnected with at least two vertices.

⟹ D is a strong split dominating set of G

$\therefore \gamma_{ss}(\bar{G}) \leq \gamma_{ss}(G)$

V.TOTAL STRONG SPLIT DOMINATION NUMBER OF A GRAPH

THEOREM 5.1

If diam (G) ≤ 3, then $\gamma_{sns}(G) \leq p - m$

Where m is the number of cut vertices of G.

Proof:

Case (i): If G has no cut vertices

Then $\gamma_{sns}(G) \leq p$

Case (ii): Suppose G has cut vertices.

Let S be the set of all cut vertices with $|S| = m$

Let u, v ∈ S Suppose u and v are not adjacent

$\Rightarrow \exists$ two vertices, u_1 and v_1 such that u_1 is adjacent to u and

v_1 is adjacent to v.

$\Rightarrow d(u_1, v_1) \leq d(u_1, u) + d(u, v) + d(v, v_1)$

$d(u1, v_1) \leq 4$

which is a contradiction to diam (G) ≤ 3

Hence every two vertices in S are adjacent.

$\Rightarrow \langle S \rangle$ is complete and every vertex in S is adjacent to at least one vertex in

V – D

$\Rightarrow$ V – S is a strong split dominating set of G

$\Rightarrow \gamma_{sns}(G) = |V - S|$

$\therefore \gamma_{sns}(G) \leq p - m$

THEOREM 5.2

For any cycle C_n with $n \geq 6$ vertices $\gamma_{tss}(C_n) = f(x) = \begin{cases} \frac{2n}{3} i\text{ı } n \equiv 0(m \quad 3) \\ 2n + \frac{1}{3} i\text{ı } n \equiv 1(m \quad 3) \end{cases}$

Proof:

Let $V(C_n) = \{v_0, v_1, v_2, v_3, \ldots, v_{n-1}\}$ be the vertex set of the cycle C_n

Let D be the total strong split dominating set of C_n

Consider the sets, $D_1 = \{ v_{3i}, v_{3i+1} / i = 0, 1, 2, \ldots, \}$ when

D_2 = { v_{3i}, v_{3i+1} / i = 0, 1, 2, . . . , } ◻ { v_{n-2} }

The above two sets achieve the total strong split property of C_n in the respective parity conditions.

REFERRENCES

1. F.Harary, Graph Theory. Addison – Wesley, Reading mass (1969).
2. T.W.Haynes, S.T.Hedetniemi and P.J.Salater (1998): "Fundamental of domination in graphs", Marel Dekker, Inc., New York.
3. V.R.Kulliand B.Janakiram (2000): "The split domination number of a graph", Indian J.Pure appl. Math., (31): 545 -550
4. V.R.Kulli and B.Janakiram (1999): "The Strong split domination number of a graph", Gulburga university, Dept. of Mathematics, Report (1999).
5. V.R.Kulli and B.Janakiram (2003): "The Strong split domination number of a graph", Gulburga university, Dept. of Mathematics, Report (1999).
6. V.R.Kulli and B.Janakiram (1997): "The split domination number of a graph", Graph Theory Notes of New York academy of sciences XXXII.P.16-19.

