MULTIPLE MAGIC GRAPHS WITH 2 – VERTEX AND 3 – VERTEX

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Abstract: **Let us take a graph G in a size of m and in the order of n. The vertex label and also the labels on the perimeters incident at that vertex, augment to a fixed constant called the magic constant of G at every vertex. If the label set of the vertices is $\{a, 2a, \dots\dots, na\}$, this labeling is $a-$ vertex multiple magic. Properties of $a-$ vertex multiple magic graphs are presented in this article.**

Keywords: $a-$ vertex multiple magic graphs.

1. INTRODUCTION

Graphs are generally finite, simple and undirected. A graph G has vertex set $V = V(G)$ and the edge set E= E(G). We let $n = |V|$ and $m = |E|$. N(u) is a notation of the set of nearby parts of a vertex v. A total labeling of G is a bijection $f : V \cup E$ →{1,2,3,4……,m+n} and the associated weights of a vertex v_i in G is $w_f(v_i) = f(v_i) + \sum f(v_i v_j)$.

Graph theory was introduced by Leonhard Euler in 1736 once he began discussing whether or not it had been potential to cross all of the bridges within the town of Kaliningrad, Russia just the once. The publication of the matter and his planned resolution began what's currently referred to as graph theory. Since then, the subject has been completely studied by mathematicians like Thomas Pennyngton Kirkman and William Sorbus aucuparia Hamilton. In Euler's original drawback, vertices painted locations round the town that were connected by bridges, or the perimeters of the graph.

The vertices of a graph is labeled in many alternative ways that. in our own way to label vertices is with numbers. Vertex-magic graphs square measure graphs labeled with ranges during which each vertex and its incident edges add up to a similar number. This range is termed the atomic number. One kind of graph that has fascinating vertex-magic properties is that the cycle graph.

The total labeling f of G is vertex magic, when each vertex has the same weight. In this case, $w\_f(v\_i) = k$. Sedlacek has started the magic labeling of graphs. $f\ (V) = \{a+1, a+2, \dots., a+n\}$, where $a \in 0,1,2 \dots, m\}$.

Even vertex magic and total labeling concept has been announced in a paper related to the consecutive vertex magic graphs. If $f\big(V(G)\big) = \{2,4,6, \dots ,2n\}$, the even vertex magic total labeling will be available

A total labeling is named edge-magic, if the total of the labels of a foothold and it's 2 endpoints equals a continuing, freelance of the chosen edge. it's referred to as vertex-magic, if the total of all labels related to a vertex is that the same, freelance of the vertex. Finally, there are few graphs, that are each edge- and vertex-magic. These graphs are referred to as wholly magic and shall be studied during this work

If a an even vertex magic labeling is admitted by a graph G, which will be called as an even vertex magic graph. The odd vertex magic total labeling is introduced by the even vertex magic graph. If $f\big(V(G)\big) = \{1,3,5,\ldots,2n-1\}$, the vertex magic total labeling will be odd. A graph G is called odd vertex magic graph if it admits an odd vertex magic labeling.

2. MULTIPLE MAGIC GRAPHS

Definition:

If the set of the labels of the vertices is $f\ (V)\ = \{a, 2a, 3a, \ldots, na\}, where\ \ a \in \{1,2,3,\ldots,[\frac{m+n}{n}]\}$. $a -$ vertex multiple magic is a graph which admits an $a -$ vertex multiple magic total labeling.

Theorem:

Let G be an $a -$ vertex multiple magic graph then the magic constant k is given by

$$K\ = 2m + n + 1 + \frac{m(m+1)}{n} - \frac{a(n+1)}{2}$$

Proof:

Assume that G is an $a -$ vertex multiple magic graph. Let f be a $a -$ vertex multiple magic labeling of a graph G with the magic constant k.

Then $f(v) = \{a, 2a, 3a, \ldots, na\}$ with $1 \leq a \leq [\frac{m+n}{n}]$

$$k \quad = f(u)\ +\ \sum_{v \in N(u)} f(uv)\ \ \forall\ u\ \in \mathrm{V}.$$

$$nk \quad = [a + 2a + 3a + \cdots + na] + 2\{1 + 2 + 3 + \cdots + m + n\}$$

$$- 2\{a + 2a + 3a + \cdots . + na\}$$

$$= 2\{1 + 2 + 3 + \cdots + m + n\} - \{a + 2a + 3a + \cdots . + na\}$$

$$= \frac{2(m+n)(m+n+1)}{2} - \frac{an(n+1)}{2}$$

$$nk \quad = (m+n)(m+n+1) - \frac{an(n+1)}{2}$$

$$nk \quad = m^2 + mn + m + mn + n^2 + n - \frac{an(n+1)}{2}$$

$$nk \quad = m^2 + 2mn + m + n^2 + n - \frac{an(n+1)}{2}$$

$$k \quad = 2m + n + 1 + \frac{m(m+1)}{n} - \frac{a(n+1)}{2}$$

Theorem:

Let G be any 2- vertex multiple magic graph with size m and magic number k. Then the maximum degree is atmost $\Delta(G) \leq \sqrt{k-2}$.

Proof:

Let us consider a vertex v with maximum degree $\Delta = \Delta(G)$.

The magic number

$$k = \ \omega(v)$$

$$\geq\ 2 + 1 + 3 + 5 + \cdots + 2\Delta - 1$$

$$\Delta \leq \sqrt{k-2}.$$

Example:

Let G be the regular graph with 6 vertices and degree 4. Show that G is 2 – vertex multiple magic graph with magic constant $k = 50$

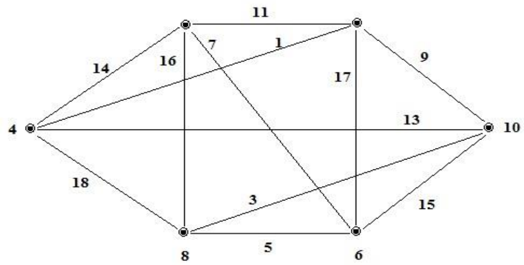

Figure: 2-Vertex multiple graph n =6, m = 12, k = 50

2 – Vertex:

Let G be the regular graph with 6 vertices and degree 5. Show that G is 2 – vertex multiple magic graph with magic constant k = 70

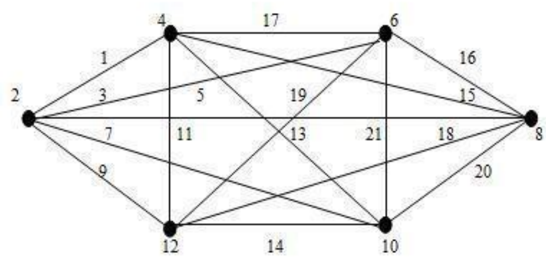

Figure: 2-Vertex multiple graph n =6, m = 15, k = 70

n = 6 {2, 4, 6, 8, 10, 12}

$$m = 15\,\{1, 3, 5, 7, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21\}$$

$$k = 2m + n + 1 + \frac{m(m+1)}{n} - \frac{a(n+1)}{2}$$

$$k = 2(15) + 6 + 1 + \frac{15\times16}{6} - \frac{2(7)}{2}$$

$$k = 30 + 7 + 40 - 7$$

$$k = 70$$

3 -Vertex:

Let G be the regular graph with 6 vertices and degree 5. Show that G is 3 – vertex multiple magic graph with magic constant k = 66.5

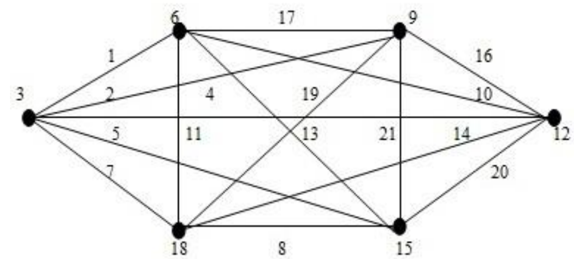

Figure: 3-Vertex multiple graph n =6, m = 15, k = 66.5

$$n = 6\,\{3, 6, 9, 12, 15, 18\}$$

$$m = 15\,\{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 21\}$$

$$k = 2m + n + 1 + \frac{m(m+1)}{n} - \frac{a(n+1)}{2}$$

$$k = 2(15) + 6 + 1 + \frac{15\times16}{6} - \frac{3(7)}{2}$$

$$k = 30 + 7 + 40 - 10.5$$

$$k = 66.5$$

3. CONCLUSION

In this paper, I have discussed about some of the basic graphs like 2-vertex and 3-vertex. We can prove for other 2-vertex and 3-vertex same types of graph with different value k which satisfy vertex magic total labeling as well. Vertex, Edges and Diagrams are all same but k value is different.

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