# Types of Graph Coloring and Its Simple Properties

S.Senthura Devi
C.Chithirai Selvi Msc, M.phil (Maths), Asst.professor
Department of Mathematics
Ponnaiyah Ramajayam Institute of science & Technology (PRIST)
Thanjavur-613403, Tamilnadu, India

**Abstract - Graph coloring is one of the most vital ideas in graph theory and it's large variety of applications in everyday life. Varied coloring ways square measure obtainable and may be used on demand basis. the correct coloring of a graph G is that the coloring of the vertices and edges with least variety of colours such that no 2 vertices ought to have identical color. The minimum number of color is called the chromatic number χ(G) and the graph G is called properly colored graph. This paper presents the applications of graph coloring and its importance in numerous fields.**

**Keyword: Chromatic number χ(G), regular graph,cut vertex, Edge coloring, Vertex coloring.**

## 1. Introduction

A Graph G consists of try (V (G),E(G)) wherever V (G) may be a non-empty finite set whose component are vertices or points and E(G) may be a set of unordered pairs of distinct parts of V (G). Variety of vertices in a graph is termed its order and also the number of edges during a graph is termed its size. We have a tendency to denote the order and size of a graph typically by n and m severally. A graph of order n and size m is referred as a (n, m) graph. If e = ∈ E(G), the sting e is alleged to affix u and v. We have a tendency to write e = u v and that we say that the vertex u and v are adjacent.

## 2. Preliminaries

**Definition 2.1**

The degree of a vertex of a graph is that the vary of edges incident to the vertex, with loop counted double.

The degree of vertex denoted by deg(v) or deg v. In a graph G the degree of the graph is denoted by Δ (G), and then the minimum degree of graph G, denoted by d(G).

**Definition 2.2**

If a graph does not has any parallel edges and self loops, it will be called as a simple graph.

**Definition 2.3**

If the edge set of a graph is empty, it will be called as null graph.

**Definition 2.4**

A graph in which any two distinct vertices are adjacent is called a complete graph. The complete graphwith n vertices is denoted by $K_n$.

**Definition 2.5**

A graph is termed planar if one will draw the graph within the plane (or paper) such no 2 edge cross.

**Definition 2.6**

If the vertices of a graph is splitted into two disjoint sets named U and V. They are splitted such that each edge connects a vertex in U to a minimum of 1 in V. Since the U and V are the elements of the graph, vertex sets U and V unit usually brought up.

**Definition 2.7**

A clique could be a set of vertices of an afloat graph such each 2 distinct vertices within the lot areadjacent; that's, its induced sub graph is complete.

 

### 3. Graph Coloring

A special case of graph labeling is called as graph coloring in graph theory; it's academic degree assignment of labels historically noted as "colors" to parts of a graph subject to sure constraints. In its simplest type, it's some way of coloring the vertices of a graph such that no 2 adjacent vertices share constant color; this can be known as a vertex coloring. Similarly, a foothold coloring assigns a color to every edge so no 2 adjacent edges share constant color.

### 3.1. Vertex Coloring

If none of the 2 distinct adjacent vertices have constant color, the coloring will be correct.

• k- Vertex colorable:

A graph G is k- vertex colorable if G incorporates a correct k- vertex coloring.

K - Vertex colorable is additionally known as as k-colorable.

•Chromatic range $\chi(G)$:

The chromatic range $\chi(G)$ of a graph G is that the minimum k that, G is k-colorable.

• K- Chromatic:

If $\chi(G) = k$, G is claimed to be k-chromatic.

### 3.2. Edge Coloring

• k- edge coloring:

A k- edge coloring of a loop less graph G is an assignment of k colours, 1,2,…...,k, to the perimeters of G. The coloring is correct if no 2 distinct adjacent edges have constant color.

• k- edge colorable:

A graph G is k- edge colorable if G incorporates a correct k−edge coloring. Edge chromatic range $\chi'(G)$: the sting chromatic range $\chi'(G)$, of a loop less graph G is that the minimum k that G is k−edge colorable.

• k- edge chromatic:

If $\chi'(G)= k$, G is claimed to be k-edge chromatic..

### 4. Some Properties and Theorems

Let $v_1, v_2, \ldots, v_n$ be some ordering of V (G). For i from 1 to n, greedily assign to $v_i$ the lowest indexed color not yet assigned to lower-index neighbor of $v_i$.

This coloring is called the greedy coloring with respect to the ordering.

**Theorem 4.1**

If G is a simple graph, then

$$\chi'(G) \leq \Delta(G)+1$$

**Proof**

Let $\Delta = \Delta(G)$. With at most $\Delta+1$ colors, we shall start from the empty coloring, consider any uncolored edge (u,v) at each step, find a color for (u,v) while possibly altering the current coloring. Repeat the process until all edges are colored.

Suppose (u,v) is not yet colored with the current coloring.

Let S be the set of $\Delta+1$ colors we are going to use. Since the maximum degree is $\Delta$, not all colors appear at any particular vertex.

If there is a color not appearing at both u and v, then we can use that color

for (u ,v). Otherwise, suppose color $c_0$ does not appear at u, and $c_1$ does not appear at V. Note that $c_1 \neq c_0$. Also, let $v_0 = v$.

There must be a neighbor $v_1$ of u such that $(u, v_1)$ is colored $c_1$. Let $c_2$ be a color missing at $v_1$. If $c_2$ is also missing at u, then we can "down-shift" the colors from $v_1$ by coloring $(u, v_1)$ with $c_2$, $(u, v_0)$ with $c_1$.

 

This process cannot continue forever, hence either we can down-shift from

some $v_k$ and find a color for (u,v), or there is a smallest index $k \geq 1$ such that a color missing at $v_k$ appeared earlier in the list $c_1,...,c_{K-1}$, say $.c\iota$

If $c_0$ is missing at $v_k$, then we can color (u, $v_k$) with $c_0$, and shift the colors down. Otherwise, let P be a maximal path starting from $v_k$ which alternate colors between $c_0$ and $c\iota$.

If P goes $v_k$,………......,$v\iota$,u, then we can switch the two colors on P, and

down- shift from $v\iota$. If P goes $v_k$,………....,$v_{l-1}$(and must stop there at color $c_0$, since $c\iota$ is missing at $v_{l-1}$), then we switch colors on P, assign (u,$v_{l-1}$) with color $c_0$, and shift down from $v_{l-1}$. Lastly, if P does not touch u, $v\iota$, or $v_{l-1}$, plus the fact that P cannot end

at $v_k$, then we can switch colors on P, assign (u, $v_k$) with $c_0$, and down-shift from $v_k$.

**Theorem 4.2**

Let G be a regular graph with cut-vertex. Then show that $\chi'(G) > \Delta(G)$.

**Proof**

Suppose that G is k-regular and that vertex $v \in V(G)$ is a cut-vertex.

Paste Article Duplication Processing Re-Write Suggestion Done (Unique Article)

Furthermore, assume that we've a correct k-edge-coloring of E (G) and

thus $\chi'(G) = k = \Delta(G)$. Vertex v has k edges incident with it. These edges are coloured one through k.

Suppose that the sting coloured i has endpoints v and vi. We are going to show that for all

one $\leq i < j \leq k$, vertices vi and vjare connected by a path in $G - v$. beginning with vertex $v_i$, find a most walk whose edges are alternately coloured i and j. This path should begin with color j, since vi doesn't have a position incident thereto coloured i in $G - v$. The walk will never come back to a antecedently encountered vertex, otherwise we'd have 2 edges coloured constant that are incident to constant vertex, which might contradict the idea that our edge-coloring is correct.

And finally, every edge coloured j leads us to a vertex that's incident with a position coloured i. this is often because of the actual fact that we have a tendency to started at $v_i$, the sole vertex that's not incident with a position coloured i, and that we grasp we will ne'er come back to vertex $v_i$ . Therefore, our walk ends once we reach a vertex that doesn't have a position incident with it that's coloured j.

But vj is that the solely such vertex. so our most walk should be a vi , vj-walk.

Therefore, any vertex within the element of $G - v$ containing vi is connected to any vertex within the element of $G-v$ containing $v_j$. Since this is often true for all i and j, we have a tendency to conclude that $G-v$ has constant variety of parts as G. however this contradicts the idea that v was a cut-vertex. Therefore, if G may be a k-regular graph with a cut-vertex then $\chi'(G) > k = \Delta(G)$, as secure.

## 5. Conclusion

Graph coloring enjoys several sensible applications also as theoretical challenges. The ultimate aim of this paper is to present the importance of varied styles of coloring. An outline is given particularly to project the applications of graph coloring in Graph theory.

## References


[1] Dr.Natarajan Meghanathan, Review of Graph Theory Algorithms, Depart of Computer Science, Jackson State Univ, Jackson, MS 39217.

[2] Shirinivas. S.G., Vetrivel S. and Elango N.M., Application of Graph theory in Computer Science an overview, Int. Jornl of Engg. Science and Tech, (2010).

[3] https://en.wikipedia.org/wiki/Graph coloring.

[4] Preeti and Omprakash, Vertex-edge coloring techniques, Int. Jornl of Core Engg. & Management, 1(2) (2014), 27-32.

[5] G.J.Chaitin, Register allocation and spilling through graph coloring, SIGPLAN Symp. on Compiler Construction, (1982), 98-105.

 