3-difference cordial labeling of some tree graphs

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Abstract

Let G be a (p,q) graph. Let $f:V(G)\rightarrow\{1,2,\dots,k\}$ be a map wherever k is Associate in Nursing whole number 2≤k≤p. for every edge ultraviolet radiation assign the label $|f(u)-f(v)|$. f is termed k-difference cordial labeling of G if $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$ wherever $v_f(x)$ denotes the quantity of vertices labeled with x, $e_f(1)$ and $e_f(0)$ severally denotes the amount of edges tagged with one and not tagged with one. A graph with a k-difference cordial labeling is termed a k-difference cordial graph. during this paper we have a tendency to investigate the 3-difference cordial labeling behavior of some caterpillar

Keywords: lob star, coconut tree, caterpillar

1. Introduction

Graphs thought of here ar finite, afloat and easy. The graph labeling was initial introduced by Rosa [1] within the name of sleek labeling. A connected acyclic graph is termed a tree. In this paper we investigate the 3-difference cordial labeling behavior of X-tree, Y-tree, coconut tree and some caterpillar and lob star. Terms not outlined here are employed in the sense of Harary [2]. This can embody developing the basic tools used by mathematicians, like maths and calculus, describing multi-dimensional house, or higher understanding the philosophical meaning of arithmetic and numbers themselves. Mathematics is that the appliance of mathematical techniques to elucidate real-world systems and solve technologically relevant problems. this could embody the mechanics of a moving body, the statistics governing the atoms throughout a gas or developing extra economical algorithms for procedure analysis. These ideas unit closely connected with those of theoretical physics. Arithmetic depends on synthesis though' man's initial experience with arithmetic was of associate inductive nature. We tend to tend to elaborate in simple terms that the deductive system involves four things.

Eighth order boundary price problems are notable to arise inside the arithmetic, physics and engineering sciences. This instability may even be sculptured by Associate in Nursing eighth order normal equation with acceptable boundary conditions. For tons of dialogue concerning the eighth-order boundary price problems, see and additionally the references in this. Analysis throughout this direction may even be thought-about in its early stages. Theorems that list the conditions for the existence and singularity of solutions of such problems unit contained throughout a comprehensive survey by Agarwal.
The boundary price problems with higher order square measure investigated as a result of every of their mathematical importance and additionally the potential for applications in mechanics and hydromagnetic stability. Finite-difference methodology was utilised in to hunt out the solution of eighthorder boundary price problems. Twizell et al. put together resolved another higher-order problems and encountered constant deficiencies. The divergent results unit because of the use of lower-order take a glance

at performs inside the spline ways. We've got a bent to use the variational iteration methodology (VIM) to hunt out solutions of eighth-order boundary price problems

Nonlinear equation, and wave-like equation in finite and limitless domains. The manoeuvre has been tested by many authors to be reliable and economical for an outsized style of scientific applications, linear and nonlinear what is more, perturbation technique, etc. the manoeuvre provides quickly merging consecutive approximations of the precise resolution if such a solution exists; otherwise variety of approximations could also be used for numerical functions. the manoeuvre is effectively used in and thus the references in this. The perturbation technique suffers from the procedure work, notably once the degree of nonlinearity can increase. Moreover, the Adomian technique suffers from the delicate algorithms used to calculate the Adomian polynomials that ar necessary for nonlinear problems.

2. K - Difference cordial labeling

DEFINITION 1. Let G be a (p,q) graph. Let $f: V(G) \to \{1,2,\dots,k\}$ be a map wherever k is associate number 2≤k≤p. for every edge actinic ray assign the label $|f(u) - f(v)|$. f is termed k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ wherever $v_f(x)$ denotes the quantity of vertices labeled with x, $e_f(1)$ and $e_f(0)$ severally denotes the quantity of edges labeled with one and not tagged with one. A graph with a k-difference cordial labeling is termed a k-difference cordial graph.

Let CT_n.let V(CT_n)={ u_i, v_j : $1 \leq i \leq n$, $1 \leq j \leq n-2$ } and E(CT_n)= {$u_i u_{i+1}$: $1 \leq i \leq n-1$ } ∪ {$u_1 v_i$: $1 \leq i \leq n-2$ }. Then CT_n is called the coconut tree.

THEOREM. 2.1

The coconut tree C_n is 3-difference cordial

Proof:

Let G=C_n.let V(G)={ u_i, v_j : $1 \leq i \leq n$, $1 \leq j \leq n-2$ } and E(G)={$u_i u_{i+1}$: $1 \leq i \leq n-1$ } ∪ {$u_1 v_i$: $1 \leq i \leq n-2$ }.

Case1: $n \equiv 0 (mod\ 3)$

First of all we consider the vertices u_i. Assignment of the value of the label 1,3,2 to the vertices u_1, u_2, u_3 respectively. Then we tend to assign the label 1,3,2 to consequent 3 vertices u_4, u_5, u_6 severally. Continued like this, we tend to assign the label to consequent 3 vertices and then on. The final vertex u_n has received the so called label by the number a pair of. Next to that, the attention has been moved to the vertices v_j. Fixing the label one to the final vertex. Assign the label 1,2,3 to the most important vertices v_2, v_3, v_4 severally. Then we have a gorgeous tendency to tend to put the assignment of the label 1,2,3 to resulting three vertices v_5, v_6 , $v_7$severally. Continued like this, we have a big tendency to tend to assign the label to resulting three vertices so on. So the last vertex v_{n-2} received the label by the amount 3.

Case 2: $n \equiv 1 (mod\ 3)$

Consider the vertices u_i. Fix the label 1 to the vertex u_1. Assign the label 2,3,1 to the vertices u_2, u_3, u_4 respectively. Then we have a poor tendency to assign the label 2,3,1 to future 3 vertices u_5, u_6, u_7 severally. Continued like this, we have a killer tendency to assign the label to future 3 vertices and then on. Clearly the last vertex received the label by the quantity one. Next our we've got an inclination to maneuver to the vertices . Fix the label 2,3 to the vertices and severally. Assign the label 1,2,3 to the vertices, severally. Then we've got an inclination to assignd the label 1,2,3 to future three vertices, severally. Continued like this, we've got an inclination to assign the label to future three vertices and so on. Therefore the last vertex received the label by the quantity 3

Case 3: $n \equiv 2(mod\ 3)$

First we tend to contemplate the vertices u_i. Fix the label 1,3 to the vertices u_1 and u_2 severally. Assign the label 2,3,1 to the vertices u_3, u_4, u_5 severally. Then we tend to assign the label 2,3,1 to successive 3 vertices u_6, u_7, u_8 severally.

Table 2.1. Vertex and edge condition

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0(mod\ 3)$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$
$n \equiv 1(mod\ 3)$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$n \equiv 2(mod\ 3)$	$\frac{2n-1}{3}$	$\frac{2n-4}{3}$	$\frac{2n-1}{3}$

Table 2.2. Vertex and edge condition

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0,2(mod\ 3)$	n-1	n-2
$n \equiv 1(mod\ 3)$	n-2	n-1

Continuing like this, we tend to assign the label to successive 3 vertices then on. So the last vertex u_n received the label one. Next we tend to move to the vertices v_j. Assign the label 1,2,3 to the vertices v_1 , v_2 v_3 severally. Then we tend to assign the label 1,2,3 to successive 3 vertices v_4 , v_5 , v_6 severally. Continued this manner, we tend to assign the label to successive 3 vertices then on. Clearly the last vertex received the label by the whole number three. So the vertex and edge conditions are given by the tables 2.1 and 2.2

THEOREM 2.2

Let G be a graph with V(G)={ u_i: $1 \le i \le n$} ∪ { $v_i^{(j)}$, $1 \le i \le 6$, $1 \le j \le n$} and E(G)={ $u_i u_{i+1}$, $1 \le i \le n-1$} ∪ { $u_i v_j^{(i)}$: $1 \le i \le n$, $1 \le j \le 6$}.Then G is 3-difference cordial. The theorem has 6 cases.

Proof:

Case 1: $n \equiv 0(mod\ 6)$

First we consider the path vertices u_i. Assign the label 1,2,2,3,2,1 to the first six vertices u_1, u_2 u_3, u_4, u_5, u_6 respectively. Then we tend to assign the label 1,2,2,3,2,1 to ensuing six vertices u_7, u_8, u_9, u_{10} , u_{11} , u_{12} severally. Continued like this, we tend to assign the label to ensuing six vertices then on. Clearly the last vertex u_n received the label by the number one. Next our attention move to the vertices $v_i^{(j)}$. Assign the label 1,2,3,1,3,3 respectively to the vertices $v_1^{(1)}$, $v_2^{(1)}$, $v_3^{(1)}$, $v_4^{(1)}$, $v_5^{(1)}$, $v_6^{(1)}$.

The label 1,2,3,1,2,3 are assigned to the pendent vertices which are adjacent to u_2. Similarly the labels 1,2,3,1,2,3 are assigned to the pendent vertices of adjacent to u_i, $3 \le i \le 6$. The label to the pendent vertices are adjacent to u_{6i+1} as in the pendent vertices which are adjacent to u_1, the pendent vertices are adjacent to u_{6i}, u_{6i+2}, u_{6i+3}, u_{6i+4}, u_{6i+5} as in the pendent vertices adjacent to u_2.

Case 2: $n \equiv 1(mod\ 6)$

Consider the path vertices u_i. Fix the label 1,2,2,3,1,2,3 to the first seven path vertices u_1, u_2 u_3, u_4, u_5, u_6, u_7 respectively. Assign the label 1,2,2,3,2,1 to following six path vertices u_8, u_9, u_{10} , u_{11} , u_{12} , u_{13} severally. Then we tend to assign the label 1,2,2,3,2,1 to following six vertices u_{14}, u_{15}, , u_{16} , u_{17} , u_{18} , u_{19} severally. Continuing like this, automatically we have put and assign the label to following six vertices so on. Thus the last vertex received the label by the whole number one. Next our attention of the cause of assignment and move to the vertices $v_i^{(j)}$. The vertices $v_1^{(1)}$, $v_2^{(1)}$, $v_3^{(1)}$, $v_4^{(1)}$, $v_5^{(1)}$, $v_6^{(1)}$ severally. Next we assign the label 1,2,3,1,2,3 to the pendent vertices which are adjacent to u_2. Similarly the researchers put the assignment of the label 1,2,3,1,2,3 to the pendent vertices which are adjacent to u_i, $3 \le i \le 7$. Then we are trying to assign the label 1,2,3,1,3,3 to the vertices $v_1^{(8)}$, $v_2^{(8)}$, $v_3^{(8)}$, $v_4^{(8)}$, $v_5^{(8)}$, $v_6^{(8)}$ respectively. Next we planned to assign the label to the pendent vertices on the place which are adjacent to u_{6i+2}, i=2,3,……. as in the pendent vertices which are adjacent to u_8 . Then we assign the label to the pendent vertices which are hanging adjacent to u_{6i}, u_{6i+1}, i=2,3,……….. as in the pendent vertices which are adjacent to u_1 and the place of the label dependent vertices which are adjacent to u_{6i+3}, u_{6i+4}, u_{6i+5}, i=1,2,3,……….. as in the pendent vertices which are adjacent to u_1.

Case 3: $n \equiv 2(mod\ 6)$

First we consider the path vertices u_i. Fix the label 2,1 to the path vertices u_1, u_2 respectively. Assign the label 1,2,2,3,2,1 to the path vertices u_3, u_4, u_5, u_6, u_7, u_8 respectively. Then we assign the label 1,2,2,3,2,1 to the next six vertices u_9, u_{10} , u_{11} , u_{12} , u_{13},u_{14}respectively. Continuing this manner, we have a tendency to assign the label to consequent six matrix vertices so on. Clearly the last vertex u_n. Then to future, we have a tendency to assign the label to the pendent vertices that ar adjacent to , , i=0,1,2,3,……. as within the pendent vertices that ar adjacent to and also the pendent vertices that ar adjacent to , , , i=1,2,3,……. as within the pendent vertices adjacent to . Next we have a tendency to assign the label to the pendent vertices that ar adjacent to , i=1,2,3,……….. as within the pendent vertices that ar adjacent to .

Case 4: $n \equiv 3(mod\ 6)$

we consider the path vertices u_i. Fix the label 2,2,1 to the path vertices u_1, u_2 , u_3 respectively. Assign the label 1,2,2,3,2,1 to the next six path vertices u_4, u_5, u_6, u_7, u_8, u_9 respectively. Then we assign the label 1,2,2,3,2,1 to the next six vertices u_{10} , u_{11} , u_{12} , u_{13},u_{14}, u_{15} respectively. Proceeding like this, we tend to assign the label to following six vertices and then on. Therefore, the last vertex u_n received the label by the whole number one. Next our attention move to the vertices $v_i^{(j)}$. Assign the label of the vertices 1,2,3,1,2,3 to the vertices $v_1^{(1)}$, $v_2^{(1)}$, $v_3^{(1)}$, $v_4^{(1)}$, $v_5^{(1)}$, $v_6^{(1)}$ severally. Next we assign the label 1,2,3,1,2,3 to the pendent vertices adjacent to u_2. Now assign the label 1,2,3,1,3,3 to the vertices

$v_1^{(3)}, v_2^{(3)}, v_3^{(3)}, v_4^{(3)}, v_5^{(3)}, v_6^{(3)}$ respectively. Next we assign the label is put to the pendent vertices which are adjacent to u_{6i+4}, i=0,1,2,3,……. as in the pendent vertices which are adjacent to u_3 and the pendent vertices which are adjacent to $u_{6i}, u_{6i+1}, u_{6i+2}, u_{6i+3}$, i=1,2,3,……. as in the pendent vertices adjacent to u_1. Finally we assign the label to the pendent vertices which are adjacent to u_{6i+5}, i=0,1,2,3,……….. as in the pendent vertices which are adjacent to u_1.

Case 5: $n \equiv 4(mod\ 6)$

First we consider the path vertices u_i. Fix the label 3,2,2,1 to the path vertices u_1, u_2, u_3, u_4 respectively. Assign the label 1,2,2,3,2,1 to the next six path vertices $u_5, u_6, u_7, u_8, u_9, u_{10}$ respectively. Then we assign the label 1,2,2,3,2,1 to the next six vertices $u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}$ respectively. Continuing this fashion, we have a tendency to assign the label to following six vertices and then on. Clearly, the last vertex u_n received the label by the number one. Next our attention move to the vertices $v_i^{(j)}$. Assign the label 1,2,3,1,2,3 to the vertices $v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, v_4^{(1)}, v_5^{(1)}, v_6^{(1)}$ severally. Next we assign the label to the pendent vertices which are adjacent to u_{6i+3}, u_{6i+4}, i=0,1,2,3,……. as in the pendent vertices which are adjacent to u_1 and the pendent vertices which are adjacent to $u_{6i}, u_{6i+2}, u_{6i+1}$,i=1,2,3,……. as in the pendent vertices adjacent to u_1. Finally we assign the label to the pendent vertices which are adjacent to u_{6i+5}, i=1,2,3,……….. as in the pendent vertices which are adjacent to u_5.

Case 6: $n \equiv 5(mod\ 6)$

Consider the path vertices u_i. Fix the label 2,1,2,3,1 to the path vertices u_1, u_2, u_3, u_4, u_5 respectively. Assign the label 1,2,2,3,2,1 to the path vertices $u_6, u_7, u_8, u_9, u_{10}, u_{11}$ respectively. Then we assign the label 1,2,2,3,2,1 to the next six path vertices $u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}$ respectively. Proceeding like this, we tend to assign the label to future six vertices then on. Therefore, the last vertex u_n received the label by the whole number one. Next our attention move to the vertices $v_i^{(j)}$. Assign the label 1,2,3,1,2,3 to the vertices $v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, v_4^{(1)}, v_5^{(1)}, v_6^{(1)}$ severally. Next we assign the label 1,2,3,1,3,3 to the vertices $v_1^{(6)}, v_2^{(6)}, v_3^{(6)}, v_4^{(6)}, v_5^{(6)}, v_6^{(6)}$ respectively. Then we assign the label to the pendent vertices which are adjacent to $u_{6i+2}, u_{6i+3}, u_{6i+4}, u_{6i+5}$, i=0,1,2,3,……. as in the pendent vertices which are adjacent to u_1 and the pendent vertices which are adjacent to u_{6i+1}, i=1,2,3,……. as in the pendent vertices adjacent to u_1. Therefore, the vertex and edge conditions are given in tables 2.3 and 2.4.

Table 2.3. Vertex and edge conditions

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0,3(mod\ 6)$	$\frac{7n}{3}$	$\frac{7n}{3}$	$\frac{7n}{3}$
$n \equiv 1,4(mod\ 6)$	$\frac{7n-1}{3}$	$\frac{7n+2}{3}$	$\frac{7n-1}{3}$
$n \equiv 2,5(mod\ 6)$	$\frac{7n+1}{3}$	$\frac{7n+1}{3}$	$\frac{7n-2}{3}$

Table 2.4. Vertex and edge conditions

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0,2,4(mod\ 6)$	$\frac{7n-2}{2}$	$\frac{7n}{2}$
$n \equiv 1,3,5(mod\ 6)$	$\frac{7n-1}{2}$	$\frac{7n-1}{2}$

THEOREM 2.3

Let G be a graph with V(G)={ u_i: $1 \le i \le n$} ∪ { $v_i^{(j)}$, $w_i^{(j)}$, $1 \le i \le 4, 1 \le j \le n$} and E(G)={ $u_i u_{i+1}$, $1 \le i \le n-1$} ∪ { $u_i v_j^{(i)}$: $1 \le i \le n, 1 \le j \le 4$} ∪ { $v_j^{(i)} w_j^{(i)}$: $1 \le i \le n, 1 \le j \le 4$} .Then G is 3-difference cordial.

Proof:

Case 1: $n \equiv 0(mod\ 4)$

First we consider the path vertices u_i. Assign the label 2,3,2,1 to the first four path vertices u_1, u_2 , u_3, u_4 respectively. Then we tend to assign the label 2,3,2,1 to successive four path vertices u_5, u_6 u_7, u_8 severally. Continuing like this, we tend to assign the label to successive six vertices so on. Clearly the last vertex u_n received the label by the number one. Next we tend to move to the vertices $v_i^{(j)}$. Assign the label 2,1,2,1 to the vertices $v_1^{(1)}$, $v_2^{(1)}$, $v_3^{(1)}$, $v_4^{(1)}$ severally. Then we assign the label 1,3,1,3 to the pendent vertices adjacent to u_2 and assign the label 2,3,2,3 to the pendent vertices adjacent to u_3. Next we assign the label 3,1,1,1 to the pendent vertices which are adjacent to u_4. Now we assign the label to the pendent vertices which are adjacent to u_{4i+1} , i=1,2,3…….. as in the pendent vertices which are adjacent to u_1 and the pendent vertices which are adjacent to u_{4i+2},i=1,2,3,……. as in the pendent vertices adjacent to u_2. Assign the label three to the vertices $w_1^{(1)}$, $w_2^{(1)}$, $w_3^{(1)}$, $w_4^{(1)}$ severally and that we assign the label a pair of to the vertices $w_1^{(2)}$, $w_2^{(2)}$, $w_3^{(2)}$, $w_4^{(2)}$ severally. Then we tend to assign the label one to the vertices $w_1^{(3)}$, $w_2^{(3)}$, $w_3^{(3)}$, $w_4^{(3)}$ severally and that we assign the label 2,2,3,3 to the vertices $w_1^{(4)}$, $w_2^{(4)}$, $w_3^{(4)}$, $w_4^{(4)}$ severally. Then we assign the label to the pendent vertices which are adjacent to $v_j^{(4i+3)}$, i=1,2,3,……, $1 \le j \le 4$ as in the pendent vertices which are adjacent to $v_j^{(3)}$, $1 \le j \le 4$ and assign the label to the pendent vertices which are adjacent to $v_j^{(4i)}$, i=2,3,4,…….., $1 \le j \le 4$ as in the pendent vertices which are adjacent to $v_j^{(4)}$, $1 \le j \le 4$.

Case 2: $n \equiv 1(mod\ 4)$

Assign the label to the vertices u_{n-1}, $v_j^{(n-1)}$, $w_j^{(n-1)}$ as in case 1. Now we assign the label 2 to the vertex u_n. Next we tend to assign the label 1,1,2,2 to the vertices $v_1^{(n)}$, $v_2^{(n)}$, $v_3^{(n)}$, $v_4^{(n)}$ severally. and we tend to assign the label three,3,3,1 to the vertices $w_1^{(n)}$, $w_2^{(n)}$, $w_3^{(n)}$, $w_4^{(n)}$ severally.

Case 3: $n \equiv 2(mod\ 4)$

As in case 2, assign the label to the vertes u_{n-1} , $v_j^{(n-1)}$, $w_j^{(n-1)}$. Then we assign the label 1 to the vertex u_n. Now we have a tendency to assign the label 2,1,1,3 to the vertices $v_1^{(n)}$, $v_2^{(n)}$, $v_3^{(n)}$, $v_4^{(n)}$ severally and we have a dency to assign the label 3,3,2,2 to the vertices $w_1^{(n)}$, $w_2^{(n)}$, $w_3^{(n)}$, $w_4^{(n)}$ severally

Case 4: $n \equiv 3(mod\ 4)$

Assign the label to the vertices u_{n-1} , $v_j^{(n-1)}$, $w_j^{(n-1)}$ as in case 3. Next we assign the label 3 to the vertex u_n. Then we tend to assign the label 2,1,1,1 to the verces $v_1^{(n)}$, $v_2^{(n)}$, $v_3^{(n)}$, $v_4^{(n)}$ severally and we tend to assign the label 3,3,2,2 to the vertices $w_1^{(n)}$, $w_2^{(n)}$, $w_3^{(n)}$, $w_4^{(n)}$ severally.

Therefore the vertex condition is $v_f(1) = v_f(2) = v_f(3)$ = 3n. Also the edge condition is given in table 2.5

Table 2.5. Edge Conditions

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0(mod\ 4)$	$\frac{9n}{2}$	$\frac{9n-2}{2}$
$n \equiv 1,3(mod\ 4)$	$\frac{9n-1}{2}$	$\frac{9n-1}{2}$
$n \equiv 2(mod\ 4)$	$\frac{9n-2}{2}$	$\frac{9n}{2}$

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